Quantum Loop Effects in Inflationary Cosmology

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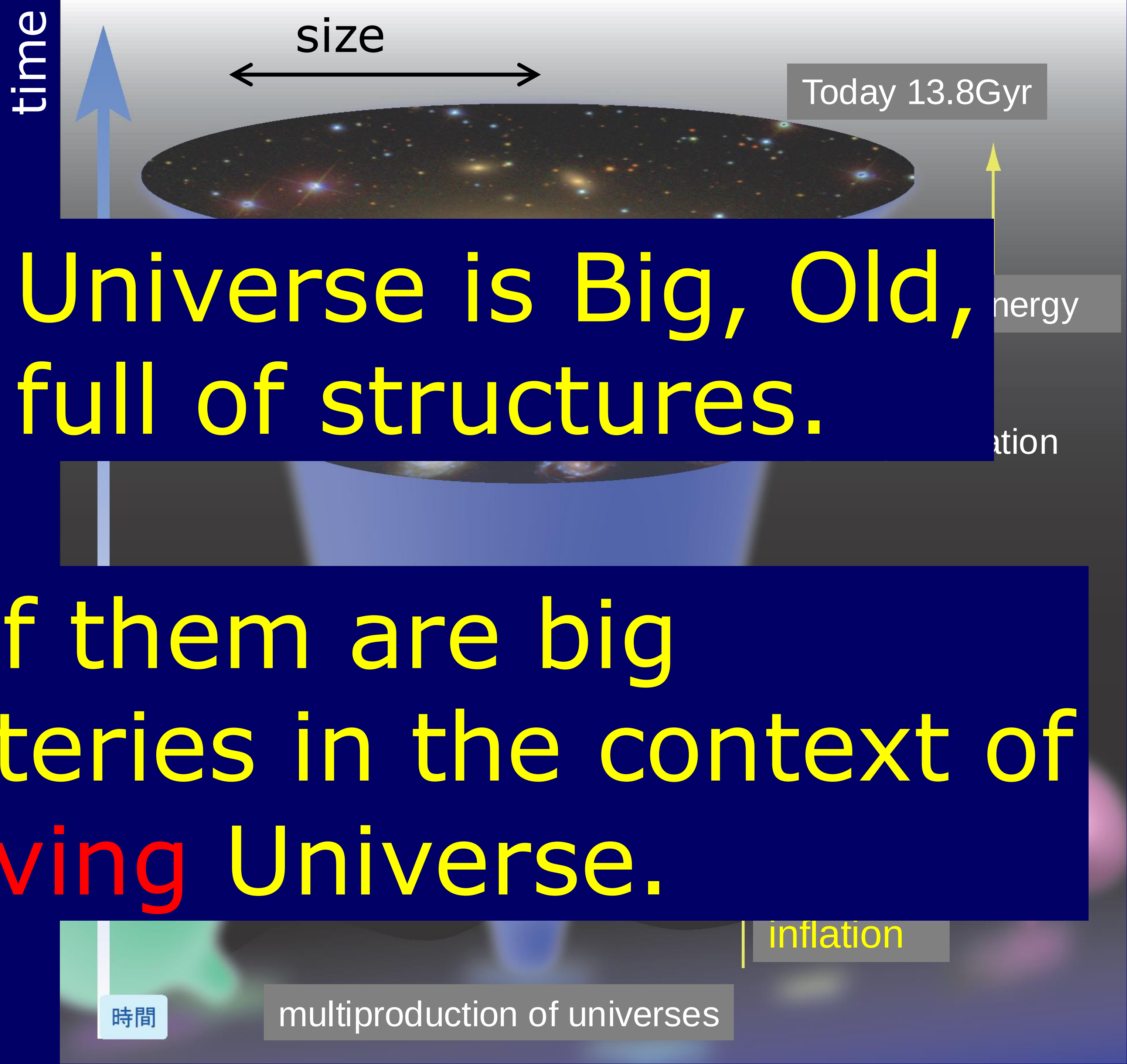


Our Universe is Big, Old, and full of structures.

All of them are big mysteries in the context of evolving Universe.

Inflation in the early Universe can solve

The Horizon Problem

Why is our Universe Big?

The Flatness Problem

Why is our Universe Old?

The Monopole/Relic Problem

Why is our Universe free from exotic relics?

The Origin-of-Structure Problem

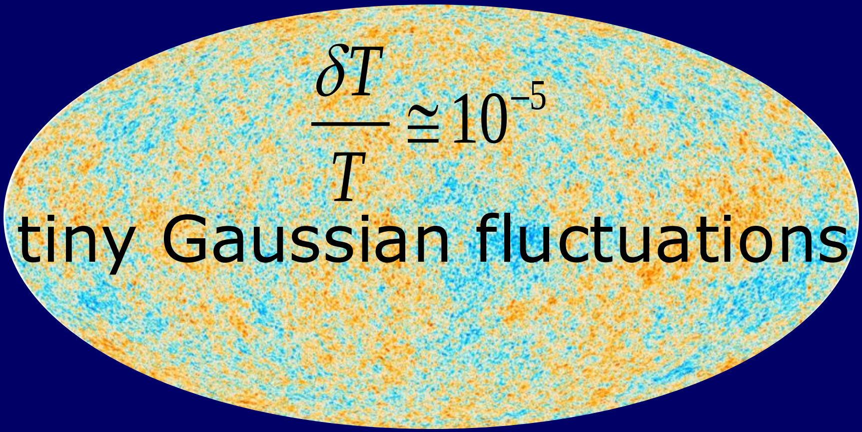
Why is our Universe full of structures?

Inflation realizes not only

Homogeneous
Background Radiation
@T=2.73K
CMB

by exponential expansion

but also



by quantum fluctuations.

What I wish to address is that considerations of quantum loop effects of perturbations greatly constrain viable class of models.

wanted to
I present two specific cases, one with non-Gaussianity and the other with formation of Primordial Black Holes.

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Constraining Primordial Black Hole Formation from Single-Field Inflation

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Why Must Primordial Non-Gaussianity Be Very Small?

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One-loop correction to the power spectrum in generic single-field inflation is calculated by using standard perturbation theory. Because of the enhancement inversely proportional to the observed red tilt of the spectral index of curvature perturbation, the correction turns out to be much larger than previously anticipated. As a result, the primordial non-Gaussianity must be much smaller than the current observational bound in order to warrant the validity of cosmological perturbation theory.

DOI: 10.1103/PhysRevLett.128.061301

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至誠依神期勝利

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Cosmological perturbation theory

Simplest canonical case $S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[M_{\rm pl}^2 R - (\partial_\mu \phi)^2 - 2V(\phi)\right].$

Incorporating comoving curvature perturbation ζ as $g_{ij} = a^2(t)e^{2\zeta}\delta_{ij}$
with $\delta\phi = 0$ and $a(t) \propto e^{Ht}$

we calculate the action for the curvature perturbation ζ to 2nd order

$$S^{(2)} = M_{\rm pl}^2 \int \mathrm{d}t\ \mathrm{d}^3x\ \boxed{a^3\epsilon}\left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i\zeta)^2\right] \qquad \varepsilon := -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2M_{pl}^2 H^2}$$

V behaves like a free massless scalar field
with a noncanonical normalization.

Introducing Mukhanov-Sasaki (MS) variable $v = z\zeta M_{\rm pl}$ with
$z = a\sqrt{2\epsilon}$, the second-order action becomes

$$S^{(2)} = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3x\left[(v')^2 - (\partial_i v)^2 + \frac{z''}{z}v^2\right]. \qquad a(\tau) \cong -\frac{1}{H\tau}$$

τ: conformal time

$$S^{(2)} = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3x\left[(v')^2 - (\partial_i v)^2 + \frac{z''}{z}v^2\right].$$

It has a canonical kinetic term, so can easily be quantized.

Mukhanov Sasaki equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0. \qquad v(x,\tau) = \int \frac{d^3k}{(2\pi)^3}\left[v_k(\tau)\hat{a}_k e^{ik\tau} + v_k^*(\tau)\hat{a}_k^\dagger e^{-ik\tau}\right]$$

$$\frac{z''}{z} = 2a^2H^2\left(1+\varepsilon+\frac{3}{2}\delta+\frac{1}{2}\delta^2+\frac{1}{2}\varepsilon\delta+\dots\right) \qquad a(\tau) = -\frac{1}{H\tau}\frac{1}{1-\varepsilon} \qquad \delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}}$$

For slow-roll inflation $\varepsilon = 1,\ \delta = 1$ we find $\frac{z''}{z} = \frac{2}{\tau^2}$

$$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right)v_k = 0.$$

$$v_k(\tau) = \frac{\mathcal{A}_k}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau} + \frac{\mathcal{B}_k}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right)e^{ik\tau}.$$

$$v_k(\tau) = \frac{1}{\sqrt{2k}}e^{-ik\tau}$$

$|k\tau| \gg 1$

$\mathcal{A}_k = 1$ and $\mathcal{B}_k = 0$ is the solution corresponding to the Minkowski mode function (vacuum) at high frequency or in the beginning.

Quantization

$$\hat{v}(\boldsymbol{k},\tau) \equiv v_{\boldsymbol{k}}(\tau)\hat{a}_{\boldsymbol{k}} + v^*_{\boldsymbol{k}}(\tau)\hat{a}^\dagger_{-\boldsymbol{k}}$$

Since v has a canonical kinetic term, conjugate momentum is simply $\hat{\pi}(\boldsymbol{k},\tau) = \hat{v}'(\boldsymbol{k},\tau)$ and the standard quantization

$$\left[\hat{v}(\boldsymbol{k},\tau), \hat{\pi}(\boldsymbol{k}',\tau)\right] = i\mathrm{h}\delta(\boldsymbol{k}-\boldsymbol{k}') \quad \Rightarrow \quad \left[\hat{a}_{\mathbf{p}}, \hat{a}^\dagger_{-\mathbf{q}}\right] = (2\pi)^3\delta^3(\mathbf{p}+\mathbf{q})$$

can be done with the normalization $v'^*_k v_k - v'_k v^*_k = i.$

Curvature perturbation:

From $\varsigma = \dfrac{v}{M_{pl}z} = \dfrac{v}{M_{pl}a\sqrt{2\varepsilon}}$, we find the mode function

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\varepsilon}}\right)_\star \frac{e^{-ik\tau}}{k^{3/2}}(1+ik\tau) \ ,$$

where $\star$ denotes horizon crossing condition $\tau = -1/k$.

Quantization

$$\hat{\zeta}(\boldsymbol{k},\tau) \equiv \frac{\hat{v}(\boldsymbol{k},\tau)}{M_{pl} a \sqrt{2\varepsilon}} = \varsigma_{\boldsymbol{k}}(\tau)\hat{a}_{\boldsymbol{k}} + \varsigma^{*}_{\boldsymbol{k}}(\tau)\hat{a}^{\dagger}_{-\boldsymbol{k}} \qquad \left[\hat{a}_{\mathbf{p}}, \hat{a}^{\dagger}_{-\mathbf{q}}\right] = (2\pi)^3\delta^3(\mathbf{p}+\mathbf{q})$$

Vacuum expectation value yields power spectrum $\varsigma(\boldsymbol{p}) \equiv \hat{\zeta}(\boldsymbol{p},\tau)$

$$\langle\zeta(\mathbf{p})\zeta(\mathbf{q})\rangle_{(0)} =: (2\pi)^3\delta^3(\mathbf{p}+\mathbf{q})\,\langle\!\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\!\rangle_{(0)} \qquad \langle\!\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\!\rangle_{(0)} = |\zeta_p(\tau)|^2$$

In the superhorizon regime $-k\tau = 1$, vacuum fluctuation is constant and given by

$$\Delta_s^2\left(r = \frac{2\pi}{k}\right) := \left\langle\!\left\langle \varsigma(\boldsymbol{k})\varsigma(-\boldsymbol{k})\right\rangle\!\right\rangle \frac{4\pi k^3}{(2\pi)^3} = \left|\varsigma_{\boldsymbol{k}}\right|^2 \frac{4\pi k^3}{(2\pi)^3} = \frac{H^2}{8\pi^2 M_{pl}^2 \varepsilon}$$

"Classicalilzation" of curvature perturbation ζ

$$\varsigma_{\boldsymbol{k}}(\tau)=\frac{iH}{2M_{pl}\sqrt{\varepsilon k^3}}(1+ik\tau)e^{-ik\tau}\to\frac{iH}{2M_{pl}\sqrt{\varepsilon k^3}}\left[1+O\left(\left(\frac{k}{aH}\right)^2\right)\right]$$ for $k \ll a(t)H$ $\left(-k\tau=\frac{k}{aH}\right)$

$$\varsigma_{\boldsymbol{k}}^*(\tau)=-\varsigma_{\boldsymbol{k}}(\tau)$$ in the superhorizon limit

So we find $\hat{\zeta}(\boldsymbol{k},\tau)=\varsigma_{\boldsymbol{k}}(\tau)\left(\hat{a}_{\boldsymbol{k}}-\hat{a}^\dagger_{-\boldsymbol{k}}\right)$
and its conjugate momentum

$$\hat{\pi}_\varsigma(\boldsymbol{k},\tau)=(M_{pl}z)^2\hat{\zeta}'(\boldsymbol{k},\tau)=(M_{pl}z)^2\varsigma_{\boldsymbol{k}}'(\tau)\left(\hat{a}_{\boldsymbol{k}}-\hat{a}^\dagger_{-\boldsymbol{k}}\right)$$

The same operator dependence!

When the decaying mode is negligible, $\hat{\zeta}(\boldsymbol{k},\tau)$ and $\hat{\pi}_\varsigma(\boldsymbol{k},\tau)$ have the same operator dependence and apparently commute with each other.

Long-wave quantum fluctuations behave as if classical statistical fluctuations.

$\Rightarrow$

Origin of large scale structures and CMB anisotropy

More precise statements

$$\left[\hat{\zeta}(\boldsymbol{k},\tau),\hat{\pi}_{\varsigma}(\boldsymbol{k}',\tau)\right]=\left[\hat{v}(\boldsymbol{k},\tau),\hat{\pi}(\boldsymbol{k}',\tau)\right]=i\mathrm{h}\delta(\boldsymbol{k}-\boldsymbol{k}')$$ always holds.

What we find is

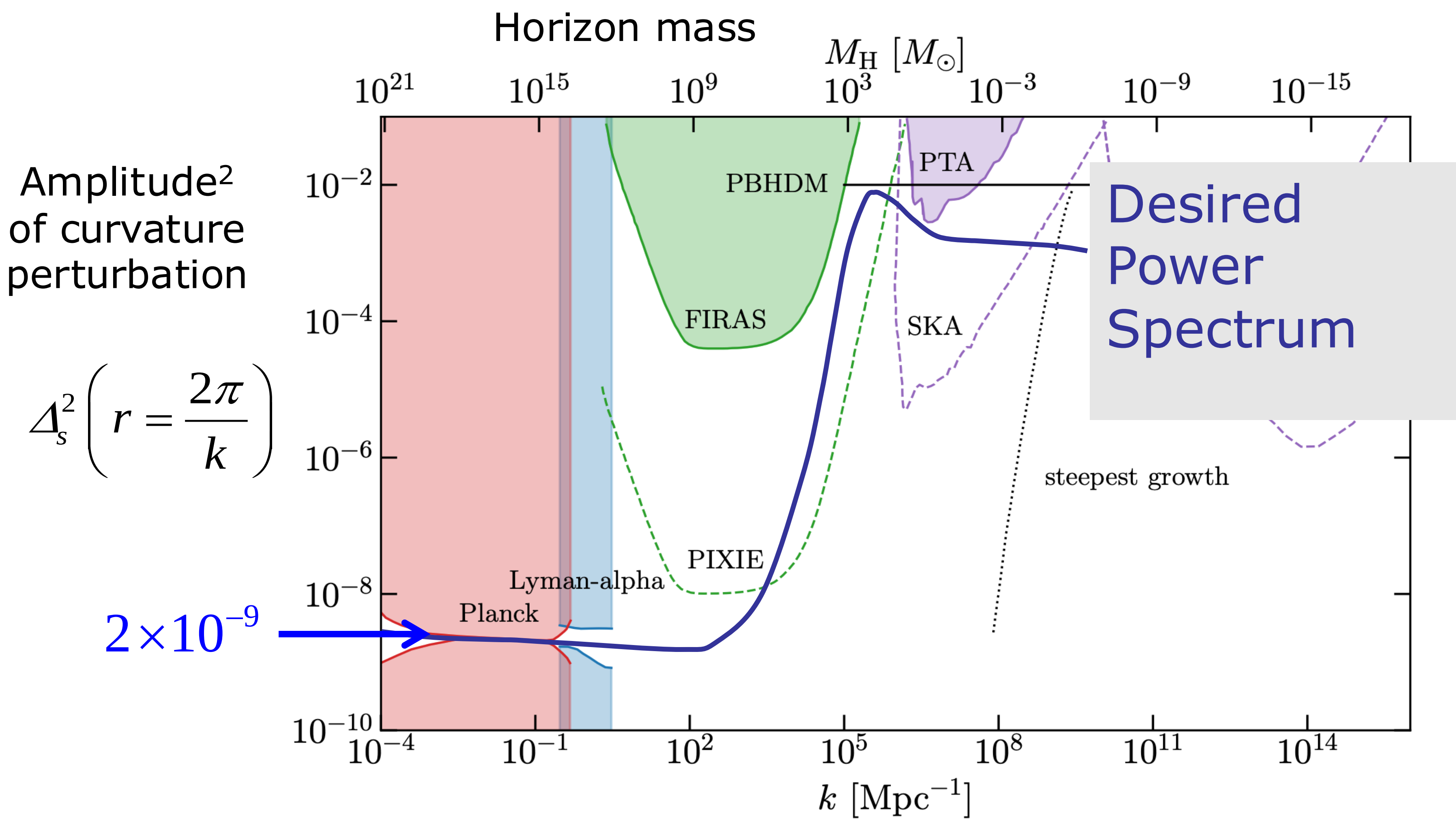
$$\left[\hat{\zeta}(\boldsymbol{k},\tau),\hat{\zeta}'(\boldsymbol{k}',\tau)\right]=\frac{1}{2M_{pl}^{2}a^{2}\varepsilon}i\mathrm{h}\delta(\boldsymbol{k}-\boldsymbol{k}')\ ]$$ decreases exponentially.

in standard slow-roll inflation with $\varepsilon \approx const =\ 1$.

In terms of the mode function

$$\zeta_k'^{*}\zeta_k-\zeta_k'\zeta_k^{*}=\frac{i}{2a^2\epsilon M_{\mathrm{pl}}^2}\quad]\quad 0$$ in standard slow-roll inflation.

If one employs an inflation model producing a peaky power spectrum on a small scale while satisfying CMB constraints, one may produce PBHs found by GWs.

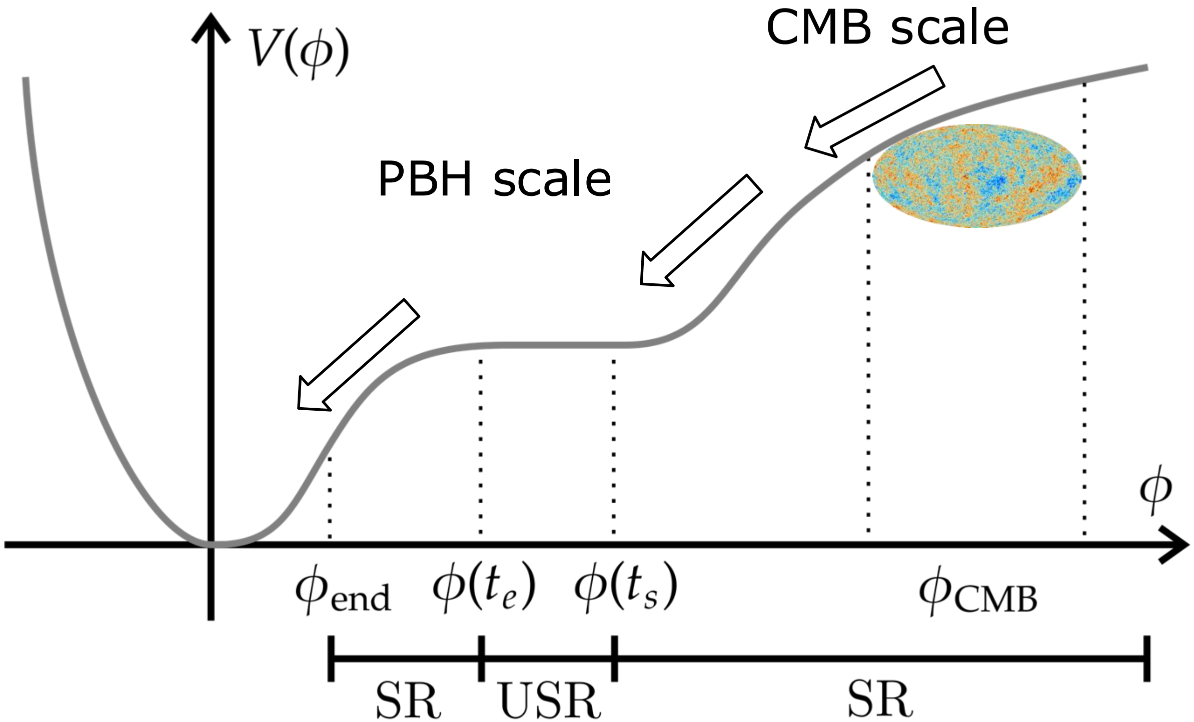


Green & Kavanagh 2007.100722

In order to realize temporal enhancement of curvature perturbation, one is tempted to adopt a model in which ε decreases temporarily.

$$\Delta_s^2\left(r=\frac{2\pi}{k}\right) := \left|\varsigma_{\boldsymbol{k}}\right|^2 \frac{4\pi k^3}{(2\pi)^3} = \frac{H^2}{8\pi^2 M_{pl}^2 \varepsilon} \; Z \qquad \Leftarrow \varepsilon \;]$$

Ivanov, Naselsky, & Novikov (1994)



SR: slow roll periods

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V[\phi]\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'[\phi] = 0$$

$$\varepsilon \approx const \equiv \varepsilon_{SR}$$

USR: ultra-slow roll period (flat potential)

$$V'[\phi] \cong 0 \qquad \ddot{\phi} + 3H\dot{\phi} = 0 \qquad \dot{\phi} \propto a^{-3}(t) \;] \qquad \epsilon = \frac{\dot{\phi}^2}{2M_{\rm pl}^2 H^2} \propto a^{-6} \qquad \Delta_s^2 \; Z$$

Ultra slow-roll (USR) inflation

Kinney (1998,2005), JY & Inoue (2002)
Martin, Motohashi, & Suyama (2013)

$$\epsilon = \frac{\dot{\phi}^2}{2M_{\rm pl}^2 H^2} \propto a^{-6}$$

Second slow-roll parameter: $\eta \equiv \frac{\&}{H\varepsilon} = -6$

In such a regime, contrary to the standard slow-roll inflation, curvature perturbation grows even on superhorizon scale, as it satisfies

$$\frac{\mathrm{d}^2\zeta_{\boldsymbol{k}}}{\mathrm{d}N^2} + (3-\epsilon+\eta)\frac{\mathrm{d}\zeta_{\boldsymbol{k}}}{\mathrm{d}N} + \left(\frac{k}{aH}\right)^2 \zeta_{\boldsymbol{k}} = 0, \qquad N = \int H dt$$

In the standard inflation with $\varepsilon, |\eta| = 1$, on superhorizon,

$\varsigma_{\boldsymbol{k}} = const$ constant mode

$\varsigma_{\boldsymbol{k}} \propto e^{-3N} = a^{-3}$ decaying mode

→ "classical" perturbation

Ultra slow-roll (USR) inflation

Kinney (1998,2005), JY & Inoue (2002)
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$$\epsilon = \frac{\dot{\phi}^2}{2M_{\rm pl}^2 H^2} \propto a^{-6}$$ ➡ Second slow-roll parameter: $\eta \equiv \frac{\dot{\varepsilon}}{H\varepsilon} = -6$

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In the standard inflation with $\varepsilon, |\eta| \ll 1$, on superhorizon,

$\varsigma_{\boldsymbol{k}} = const$ constant mode
$\varsigma_{\boldsymbol{k}} \propto e^{-3N} = a^{-3}$ decaying mode ➡ "classical" perturbation

In ultra slow-roll inflation with $\varepsilon \ll 1, \eta = -6$, on superhorizon

$\varsigma_{\boldsymbol{k}} = const$ constant mode
$\varsigma_{\boldsymbol{k}} \propto e^{3N} = a^3$ growing mode ➡ quantum nature?

Indeed we find

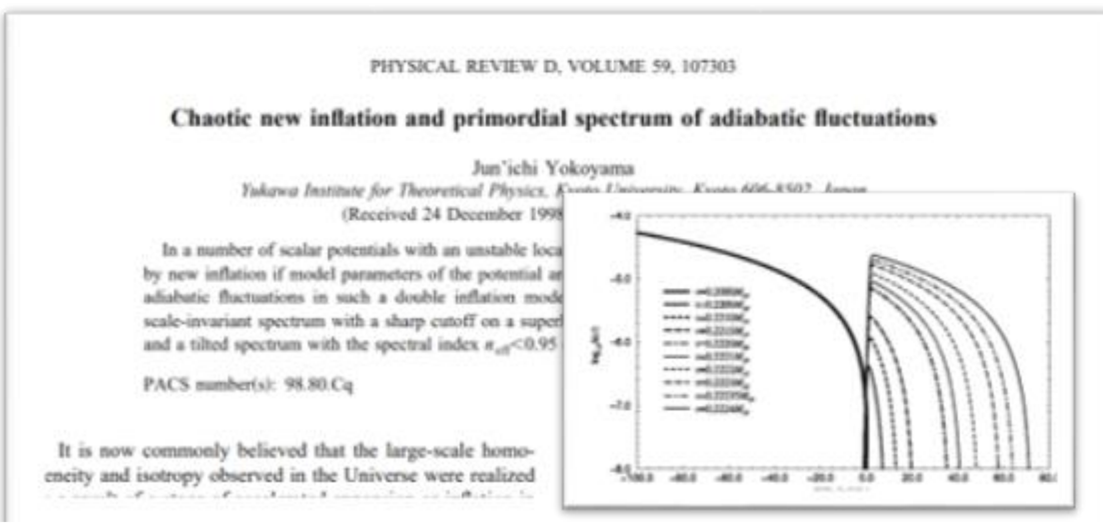
$$\left[\hat{\zeta}(\boldsymbol{k},\tau),\hat{\zeta}'(\boldsymbol{k}',\tau)\right]=\frac{1}{2M_{pl}^2a^2\varepsilon}\mathrm{i}\hbar\delta(\boldsymbol{k}-\boldsymbol{k}')\propto a^4\ \mathrm{Z}$$

$$\zeta_k'^*\zeta_k-\zeta_k'\zeta_k^*=\frac{i}{2a^2\epsilon M_{\mathrm{pl}}^2}\propto a^4\ \mathrm{Z}$$

which induces significant correction as we will see shortly.

In USR, the standard wisdom does not apply!

NB Such superhorizon growth of perturbation was also found in the chaotic new inflation model (JY 1999) and its analytic interpretation was given in (Saito, JY, Nagata 2008).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Chaotic new inflation and primordial spectrum of adiabatic fluctuations

Jun'ichi Yokoyama

PACS number(s): 98.80.Cq


Journal of Cosmology and Astroparticle Physics

Single-field inflation, anomalous enhancement of superhorizon fluctuations and non-Gaussianity in primordial black hole formation

Ryo Saito, Jun'ichi Yokoyama and Ryo Nagata

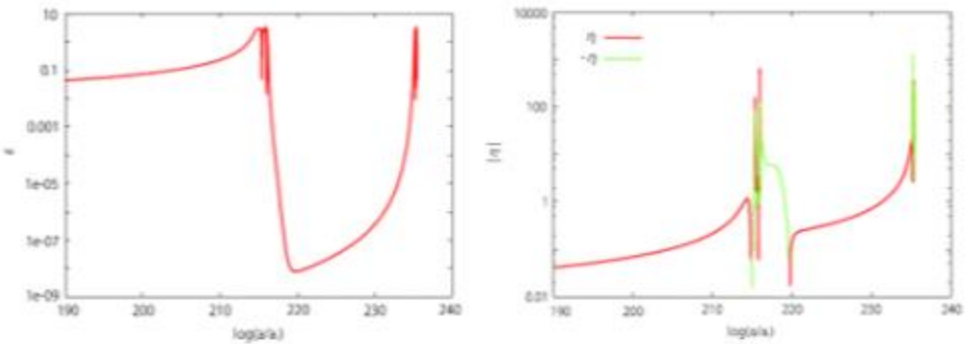

Figure 2. The evolution of slow-roll parameters ϵ (left) and η (right) with the values of the parameters $(\lambda, v) = (5.4\times 10^{-14}, 0.355\,139 M_G)$. In the right figure, the dashed portions indicate where $\eta < 0$ while the solid portions indicate where $\eta > 0$.

3.1. Evolution of curvature perturbation

Curvature perturbation in the comoving gauge ζ, in terms of which the amplitude of perturbation in the intrinsic spatial curvature of the comoving slicing $\mathcal{R}_c$ is written as

$$\mathcal{R}_c = \frac{4}{a^2}\nabla^2\zeta, \qquad (6)$$

evolves according to an equation [20]:

$$\frac{d^2\zeta_k}{dN^2} + (3-\epsilon+\eta)\frac{d\zeta_k}{dN} + \left(\frac{k}{aH}\right)^2\zeta_k = 0, \qquad (7)$$

where N is the number of e-folds and ζ_k is the Fourier transform of ζ:

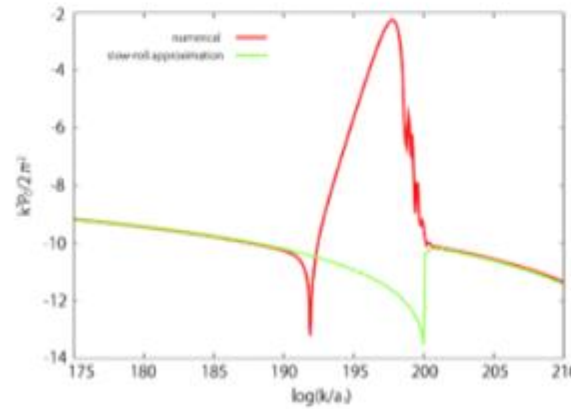

Figure 3. Power spectrum of curvature perturbation (solid line). This spectrum is calculated under the parameters $(\lambda, v) = (5.4\times 10^{-14}, 0.355\,139 M_G)$. We show also a power spectrum estimated by using the formula (10), which is used for a slow-roll inflation model (dashed line).

...orizon growth of perturbation
...odel (JY 1999) and its analytic
...2008)

Chaotic new inflation and primordial spectrum of adiabatic fluctuations

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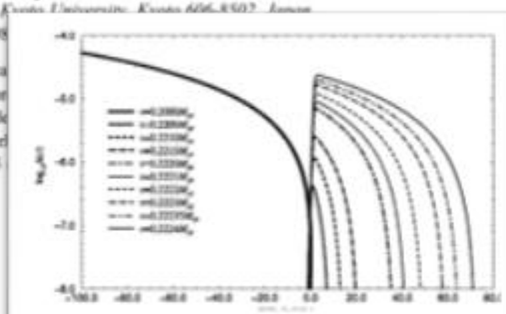
(Received 24 December 1998

In a number of scalar potentials with an unstable loca... by new inflation if model parameters of the potential ar... adiabatic fluctuations in such a double inflation mode... scale-invariant spectrum with a sharp cutoff on a super... and a tilted spectrum with the spectral index $n_{\rm eff} < 0.95$

PACS number(s): 98.80.Cq

It is now commonly believed that the large-scale homogeneity and isotropy observed in the Universe were realized



Journal of Cosmology and Astroparticle Physics
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Single-field inflation, anomalous enhancement of superhorizon fluctuations and non-Gaussianity in primordial black hole formation

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Mode function in ultra slow-roll inflation

Introducing Mukhanov-Sasaki (MS) variable $v = z\zeta M_{\mathrm{pl}}$ with $z = a\sqrt{2\epsilon}$, the mode equation reads

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0. \qquad \frac{z''}{z} = 2a^2H^2\left(1+\varepsilon+\underbrace{\frac{3}{2}\delta+\frac{1}{2}\delta^2}_{\text{cancel}}+\frac{1}{2}\varepsilon\delta+\ldots\right)$$

$$a(\tau) = -\frac{1}{H\tau}\frac{1}{1-\varepsilon} \qquad \delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}}$$

For slow-roll inflation $\varepsilon \ll 1,\ \delta \ll 1$ we find $\dfrac{z''}{z} = \dfrac{2}{\tau^2}$

For ultra slow-roll inflation we find $\delta \equiv \dfrac{\ddot{\phi}}{H\dot{\phi}} = -3$

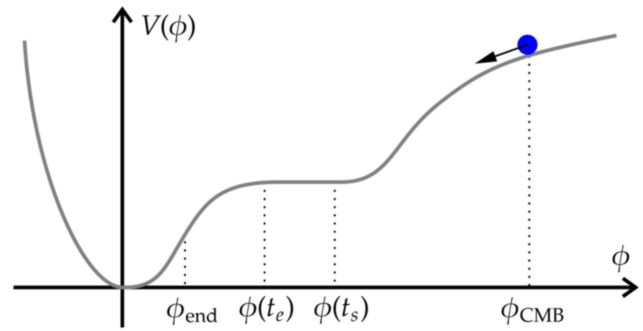
again $\dfrac{z''}{z} = \dfrac{2}{\tau^2}$

$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right) v_k = 0$ both in SR and USR regimes!

Mode function in ultra slow-roll inflation

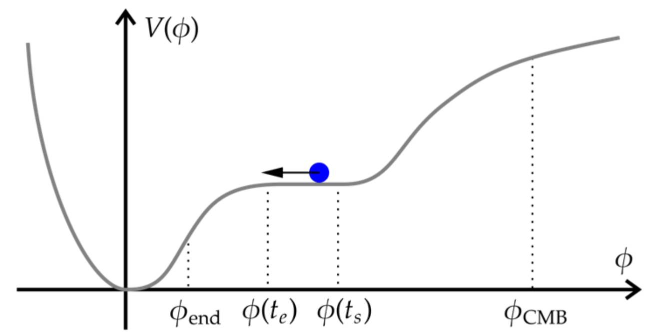
Initial slow-roll regime (CMB scale)

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \frac{e^{-ik\tau}}{k^{3/2}}(1+ik\tau)$$



Ultra slow-roll regime (PBH scale)

The mode function in this regime is found by matching ς_k and ς_k' at the transition time t_s.



$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \boxed{\left(\frac{\tau_s}{\tau}\right)^3} \frac{1}{k^{3/2}} \times \left[\mathcal{A}_k e^{-ik\tau}(1+ik\tau) - \mathcal{B}_k e^{ik\tau}(1-ik\tau)\right]$$

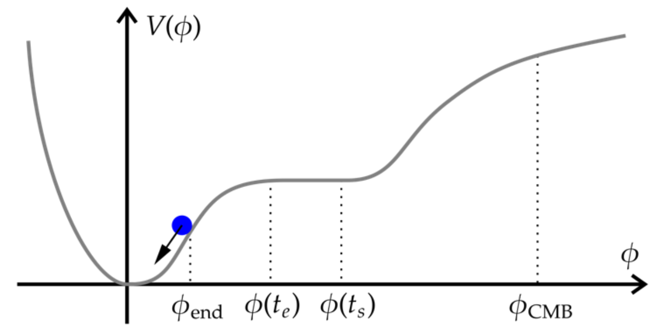
grow $\propto a^3$

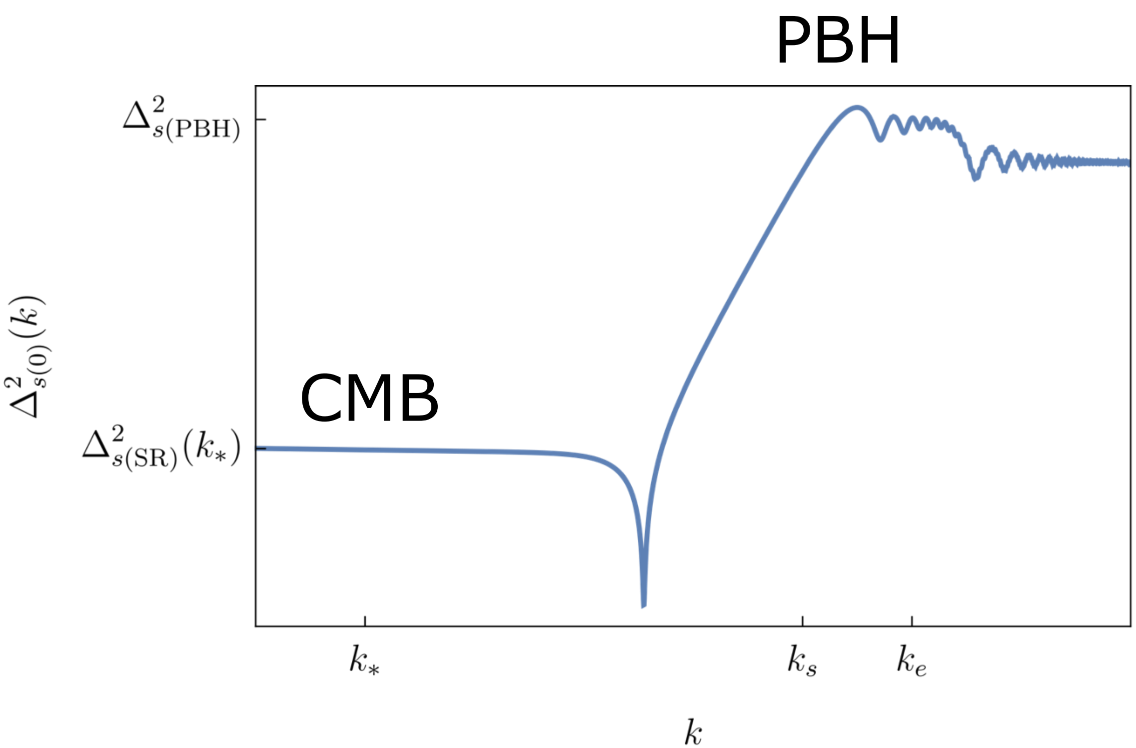
$$\mathcal{A}_k = 1 - \frac{3(1+k^2\tau_s^2)}{2ik^3\tau_s^3}, \quad \mathcal{B}_k = -\frac{3(1+ik\tau_s)^2}{2ik^3\tau_s^3} e^{-2ik\tau_s}$$

After some period of USR inflation, the system returns to SR regime again and inflation is terminated at t_0 .

At the second transition we perform similar matching again to obtain the full solution of $V_k(t)$.

Byrnes et. al. (1811.11158), Liu et. al. (2003.02075), Karam et. al. (2205.13540)





$$\Delta^2_{s(\mathrm{PBH})} \approx \Delta^2_{s(\mathrm{SR})}(k_s)\left(\frac{k_e}{k_s}\right)^6$$

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \boxed{\left(\frac{\tau_s}{\tau}\right)^3}[\ldots]$$

grow $\propto a^3$

So far we have considered only second-order action of V from the full action, which led to linear perturbation.

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[M_{\mathrm{pl}}^2 R - (\partial_\mu\phi)^2 - 2V(\phi)\right].$$

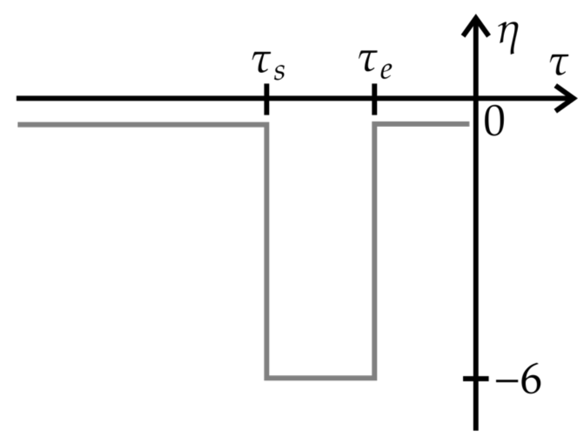
$$\Rightarrow \quad S^{(2)} = M_{\mathrm{pl}}^2 \int \mathrm{d}t\ \mathrm{d}^3x\ a^3\epsilon\left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i\zeta)^2\right]$$

Third order terms generate non-Gaussianity and one-loop correction to the power spectrum

$$\chi = \epsilon\partial^{-2}\dot{\zeta}$$

$$\Rightarrow \quad S^{(3)}[\zeta] = M_{\mathrm{pl}}^2 \int \mathrm{d}t\ \mathrm{d}^3x\ a^3\left[\epsilon^2\dot{\zeta}^2\zeta + \frac{1}{a^2}\epsilon^2(\partial_i\zeta)^2\zeta - 2\epsilon\dot{\zeta}\partial_i\zeta\partial_i\chi - \frac{1}{2}\epsilon^3\dot{\zeta}^2\zeta + \frac{1}{2}\epsilon\zeta(\partial_i\partial_j\chi)^2 + \frac{1}{2}\epsilon\dot{\eta}\dot{\zeta}\zeta^2\right]$$

The most relevant is the last term as η changes abruptly at transitions.



$$H_{\mathrm{int}}(\tau) = -\frac{1}{2}M_{\mathrm{pl}}^2\int \mathrm{d}^3x\,\epsilon\eta' a^2\zeta'\zeta^2$$

Unlike in particle physics, whose focus is transition amplitude, we wish to evaluate an expectation value or a correlation function.

$$H_{\mathrm{int}}(\tau) = -\frac{1}{2} M_{\mathrm{pl}}^2 \int \mathrm{d}^3 x \epsilon \eta' a^2 \zeta' \zeta^2$$

In-in formalism

$$\langle \mathcal{O}(\tau) \rangle = \left\langle \left[\bar{\mathrm{T}} \exp\left(i \int_{-\infty}^{\tau} \mathrm{d}\tau' H_{\mathrm{int}}(\tau') \right) \right] \hat{\mathcal{O}}(\tau) \left[\mathrm{T} \exp\left(-i \int_{-\infty}^{\tau} \mathrm{d}\tau' H_{\mathrm{int}}(\tau') \right) \right] \right\rangle$$

$\hat{\mathcal{O}}(\tau) = \zeta(\mathbf{p}_1)\zeta(\mathbf{p}_2)$: evaluated toward the end of inflation $\tau = \tau_0 \; (\longrightarrow 0)$.

Perturbative expansion

$$\langle \mathcal{O}(\tau) \rangle = \langle \mathcal{O}(\tau) \rangle_{(0,2)}^{\dagger} + \langle \mathcal{O}(\tau) \rangle_{(1,1)} + \langle \mathcal{O}(\tau) \rangle_{(0,2)}$$

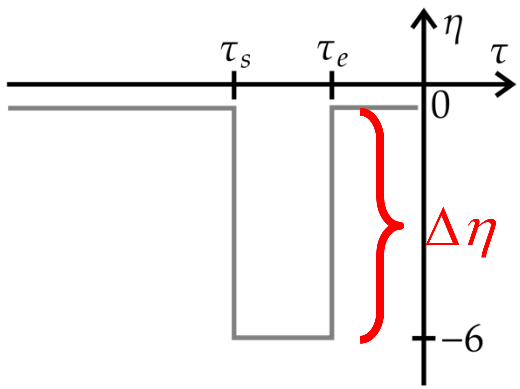
$$\langle \mathcal{O}(\tau) \rangle_{(0,2)} = -\int_{-\infty}^{\tau} \mathrm{d}\tau_1 \int_{-\infty}^{\tau_1} \mathrm{d}\tau_2 \left\langle \hat{\mathcal{O}}(\tau) H_{\mathrm{int}}(\tau_1) H_{\mathrm{int}}(\tau_2) \right\rangle$$

$$\langle \mathcal{O}(\tau) \rangle_{(1,1)} = \int_{-\infty}^{\tau} \mathrm{d}\tau_1 \int_{-\infty}^{\tau} \mathrm{d}\tau_2 \left\langle H_{\mathrm{int}}(\tau_1) \hat{\mathcal{O}}(\tau) H_{\mathrm{int}}(\tau_2) \right\rangle$$

After substituting $H_{\rm int}(\tau)$ to the perturbative expansion, we find

$$\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle_{(1,1)} = \frac{1}{4}M_{\rm pl}^4\int_{-\infty}^{0}\mathrm{d}\tau_1\ a^2(\tau_1)\epsilon(\tau_1)\eta'(\tau_1)\int_{-\infty}^{0}\mathrm{d}\tau_2\ a^2(\tau_2)\epsilon(\tau_2)\eta'(\tau_2)\int\prod_{a=1}^{6}\left[\frac{\mathrm{d}^3k_a}{(2\pi)^3}\right]\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$
$$\times\,\delta^3(\mathbf{k}_4+\mathbf{k}_5+\mathbf{k}_6)\,\langle\zeta'(\mathbf{k}_1,\tau_1)\zeta(\mathbf{k}_2,\tau_1)\zeta(\mathbf{k}_3,\tau_1)\zeta(\mathbf{p})\zeta(-\mathbf{p})\zeta'(\mathbf{k}_4,\tau_2)\zeta(\mathbf{k}_5,\tau_2)\zeta(\mathbf{k}_6,\tau_2)\rangle\,,$$
$$\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle_{(0,2)} = -\frac{1}{4}M_{\rm pl}^4\int_{-\infty}^{0}\mathrm{d}\tau_1\ a^2(\tau_1)\epsilon(\tau_1)\eta'(\tau_1)\int_{-\infty}^{\tau_1}\mathrm{d}\tau_2\ a^2(\tau_2)\epsilon(\tau_2)\eta'(\tau_2)\int\prod_{a=1}^{6}\left[\frac{\mathrm{d}^3k_a}{(2\pi)^3}\right]\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$
$$\times\,\delta^3(\mathbf{k}_4+\mathbf{k}_5+\mathbf{k}_6)\,\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\zeta'(\mathbf{k}_1,\tau_1)\zeta(\mathbf{k}_2,\tau_1)\zeta(\mathbf{k}_3,\tau_1)\zeta'(\mathbf{k}_4,\tau_2)\zeta(\mathbf{k}_5,\tau_2)\zeta(\mathbf{k}_6,\tau_2)\rangle\,.$$

Time integral is nonvanishing only at τ_s and τ_e and the latter makes a dominant contribution.



As a result, we find

$$\langle\!\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\!\rangle_{(1)} = \langle\!\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\!\rangle_{(1,1)} + 2\mathrm{Re}\,\langle\!\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\!\rangle_{(0,2)}$$
$$= \frac{1}{4}M_{\rm pl}^4\epsilon^2(\tau_e)a^4(\tau_e)(\Delta\eta(\tau_e))^2\int\frac{\mathrm{d}^3k}{(2\pi)^3}\Big[4\zeta_p\zeta_p^*\zeta_p'\zeta_p'^*\zeta_k\zeta_k^*\zeta_q\zeta_q^* + 8\zeta_p\zeta_p^*\zeta_p'^*\zeta_p\zeta_k'\zeta_k^*\zeta_q\zeta_q^*$$
$$+ 8\zeta_p\zeta_p^*\zeta_p'\zeta_p^*\zeta_k'^*\zeta_k\zeta_q\zeta_q^*$$
$$- \mathrm{Re}\left(4\zeta_p\zeta_p\zeta_p'^*\zeta_p'^*\zeta_k\zeta_k^*\zeta_q\zeta_q^* + 8\zeta_p\zeta_p\zeta_p'^*\zeta_p^*\zeta_k'\zeta_k^*\zeta_q\zeta_q^* + 8\zeta_p\zeta_p\zeta_p'^*\zeta_p^*\zeta_k'^*\zeta_k\zeta_q\zeta_q^*\right)\Big]_{\tau=\tau_e}$$

$$\boldsymbol{q}\equiv\boldsymbol{k}-\boldsymbol{p}$$

The leading term is given by

$$\langle\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\rangle_{(1)} = \frac{1}{4} M_{\rm pl}^4 \epsilon^2(\tau_e) a^4(\tau_e)(\Delta\eta(\tau_e))^2 \times 16 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left[|\zeta_p|^2 |\zeta_q|^2 \ \mathrm{Im}(\zeta_p' \zeta_p^*) \ \mathrm{Im}(\zeta_k' \zeta_k^*) \right]_{\tau=\tau_e} .$$

$$\mathrm{Im}(\zeta_k' \zeta_k^*) = \frac{i}{2}(\zeta_k'^* \zeta_k - \zeta_k' \zeta_k^*) = \frac{-1}{4a^2 \epsilon(\tau_e) M_{\rm pl}^2}$$

takes a big value at the end of USR regime as we argued.

$$\mathrm{Im}(\varsigma_k' \varsigma_k^*) = \frac{-1}{4a^2(\tau_e)\varepsilon_{SR}(a_s/a_e)^6 M_{pl}^2} = \frac{-1}{4\varepsilon_{SR} M_{pl}^2} (H\tau_e)^2 \left(\frac{\tau_s}{\tau_e}\right)^6 = -\frac{k_e^4}{k_s^6} \left(\frac{H^2}{4M_{pl}^2 \varepsilon_{SR}}\right)$$

at $\tau = \tau_e$, where we have used $k_e = a(\tau_e) H = -\dfrac{H}{H\tau_e} = -\dfrac{1}{\tau_e}$.

The leading term is given by

$$\langle\!\langle \zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\!\rangle_{(1)} = \frac{1}{4} M_{\rm pl}^4 \epsilon^2(\tau_e) a^4(\tau_e) (\Delta\eta(\tau_e))^2 \times 16 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left[|\zeta_p|^2 |\zeta_q|^2 \, \mathrm{Im}(\zeta'_p \zeta^*_p) \, \mathrm{Im}(\zeta'_k \zeta^*_k) \right]_{\tau=\tau_e} .$$

$$\mathrm{Im}(\zeta'_k \zeta^*_k) = \frac{i}{2}(\zeta'^*_k \zeta_k - \zeta'_k \zeta^*_k) = \frac{-1}{4a^2 \epsilon(\tau_e) M_{\rm pl}^2}$$

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at $\tau = \tau_e$, where we have used $k_e = a(\tau_e)H = -\frac{H}{H\tau_e} = -\frac{1}{\tau_e}$.

This is in contrast to the standard SR inflation in which $\mathrm{Im}(\zeta'_k \zeta^*_k)$ becomes exponentially small.

The leading term is given by

$$\langle\langle\zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle\rangle_{(1)} = \frac{1}{4}M_{\rm pl}^4\epsilon^2(\tau_e)a^4(\tau_e)(\Delta\eta(\tau_e))^2 \times 16\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left[|\zeta_p|^2|\zeta_q|^2\ \mathrm{Im}(\zeta_p'\zeta_p^*)\ \mathrm{Im}(\zeta_k'\zeta_k^*)\right]_{\tau=\tau_e}.$$

$$\mathrm{Im}(\zeta_k'\zeta_k^*) = \frac{i}{2}(\zeta_k'^*\zeta_k - \zeta_k'\zeta_k^*) = \frac{-1}{4a^2\epsilon(\tau_e)M_{\rm pl}^2}$$

takes a big value at the end of USR regime as we argued.

$$\mathrm{Im}(\varsigma_k'\varsigma_k^*) = \frac{-1}{4a^2(\tau_e)\varepsilon_{SR}(a_s/a_e)^6 M_{pl}^2} = \frac{-1}{4\varepsilon_{SR}M_{pl}^2}(H\tau_e)^2\left(\frac{\tau_s}{\tau_e}\right)^6 = \boxed{-\frac{k_e^4}{k_s^6}\left(\frac{H^2}{4M_{pl}^2\varepsilon_{SR}}\right)}$$

at $\tau = \tau_e$, where we have used $k_e = a(\tau_e)H = -\frac{H}{H\tau_e} = -\frac{1}{\tau_e}$.

One-loop correction

$$\Delta^2_{s(1)}(p) = \frac{1}{4}(\Delta\eta(\tau_e))^2\Delta^2_{s(\mathrm{SR})}(p)\int_{k_s}^{k_e}\frac{\mathrm{d}k}{k}\Delta^2_{s(0)}(k)$$

$$\Delta^2_{s(1)}(p) = \frac{1}{4}(\Delta\eta(\tau_e))^2\left[\Delta^2_{s(\mathrm{SR})}(p)\right]^2\left(\frac{k_e}{k_s}\right)^6\left(1.1 + \log\frac{k_e}{k_s}\right)$$

For perturbation theory to be valid, we require one loop correction << tree level (linear theory) result

$$\Delta^2_{s(1)} \ll \Delta^2_{s(\mathrm{SR})}: \quad \frac{1}{4}\underbrace{(\Delta\eta(\tau_e))^2}_{6^2=36}\underbrace{\Delta^2_{s(\mathrm{SR})}}_{2.1\times 10^{-9}}(p)\left(\frac{k_e}{k_s}\right)^6\left(1.1+\log\frac{k_e}{k_s}\right) \ll 1.$$

we obtain $\frac{k_e}{k_s} < 15$, or $\Delta^2_{s(\mathrm{PBH})} \ll 0.03\left(\frac{k_s}{k_*}\right)^{-0.03}$.

($n_s = 0.97$ at $k_* = 0.05\mathrm{Mpc}^{-1}$)

- Consider two examples that are of recent interest:
  - – PBHs as dark matter with mass $\mathcal{O}(10^{-15})M_{\odot}$ corresponding to scale $\mathcal{O}(10^{14})\mathrm{Mpc}^{-1}$ has a bound $\Delta^2_{s(\mathrm{PBH})} \ll 0.01$.
  - – PBHs as LIGO-Virgo BHs with mass $\mathcal{O}(10)M_{\odot}$ corresponding to scale $\mathcal{O}(10^{6})\mathrm{Mpc}^{-1}$ has a bound $\Delta^2_{s(\mathrm{PBH})} \ll 0.02$.

- In both cases, the upper bound contradicts with typical requirement to form a significant abundance of PBHs, which is $\Delta^2_{s(\mathrm{PBH})} \sim \mathcal{O}(0.01)$.

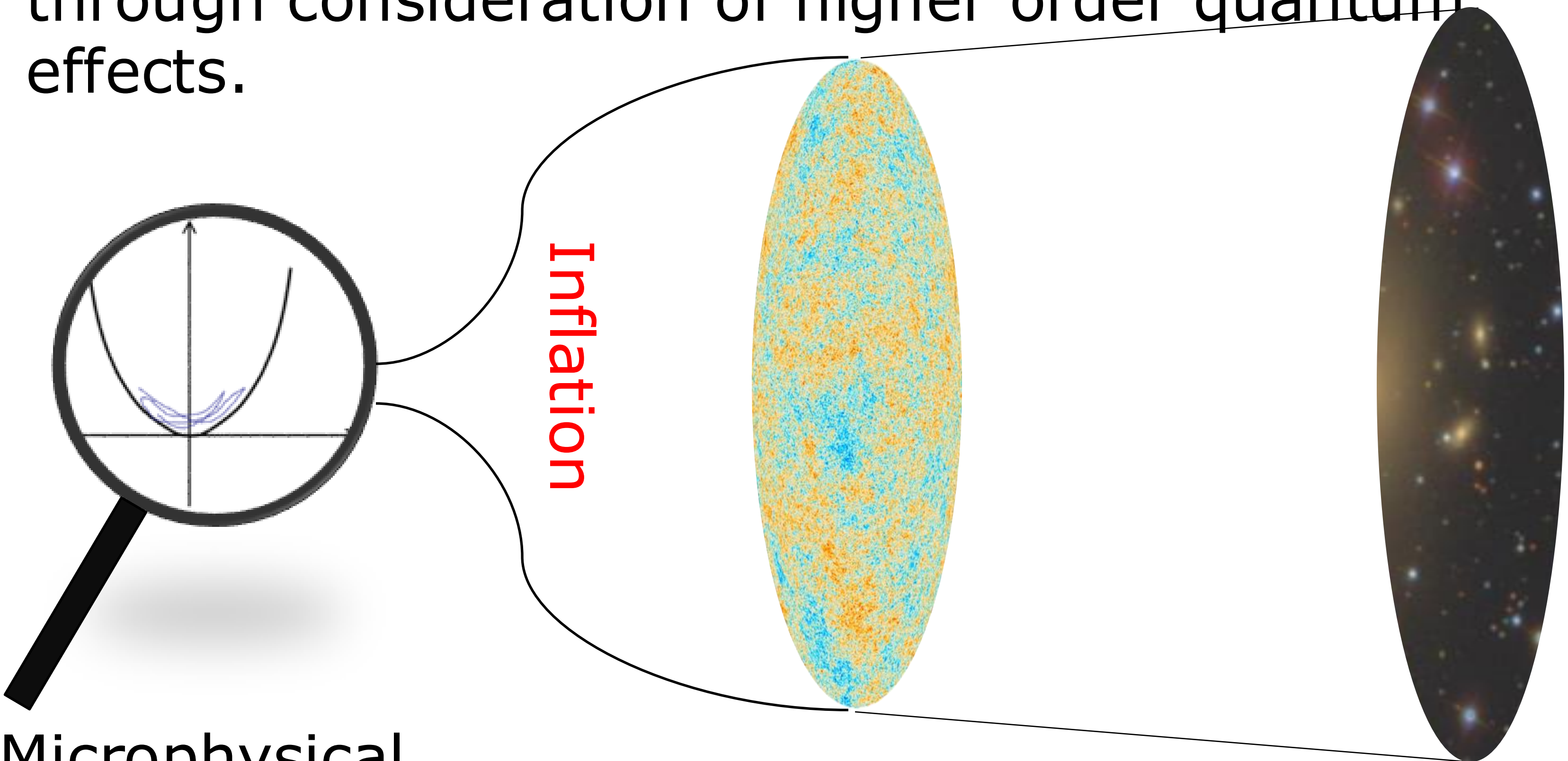
Kristiano and JY Phys Rev Lett 132, (2024)221003

A number of single-field inflation models accommodating PBH formation have the same feature, namely, sharp transition of η.

- Bump or dip: Mishra and Sahni (1911.00057)
- Upward or downward step: Cai et. al. (2112.13836), Inomata et. al. (2104.03972)
- Polynomial shape: Hertzberg and Yamada (1712.09750), Ballesteros et. al. (2001.08220)
- Chaotic new inflation with a Coleman-Weinberg potential: Saito, JY, & Nagata (0804.3470)

Conclusion

Single field inflation models predicting a large non-Gaussianity or PBH formation are in trouble through consideration of higher order quantum effects.



Microphysical zero-point fluctuations

It’s time to single out “the inflation model”!

上州無智亦無犬
剛殺伐誠易被欺
唯人立直接萬人
至誠感神期勝利
鑑

Criticism I

Our first calculation incorporated only wavenumbers leaving horizon during USR regime

$$\Delta^2_{s(1)}(p) = \frac{1}{4}(\Delta\eta(\tau_e))^2\Delta^2_{s(\mathrm{SR})}(p)\int_{k_s}^{k_e}\frac{\mathrm{d}k}{k}\Delta^2_{s(0)}(k)$$

introducing hard cutoffs to k integral.

One of the referees claimed proper UV regularization and renormalization would remove all the effects we are arguing…

I would be much surprised if this were the case, namely, if UV physics removes all nontrivial IR (yes, PBH is in the IR regime in the context of UV renormalization) effect. That would be much more interesting than what we are claiming!!

Introducing UV divergence Λ

$$\int_{k_{\mathrm{IR}}}^{k_{\mathrm{UV}}} \frac{\mathrm{d}k}{k} \Delta_{s(0)}^2(k) = \left(\int_{k_s}^{k_e} + \int_{k_e}^{\Lambda a(\tau_e)}\right) \frac{\mathrm{d}k}{k} \Delta_{s(0)}^2(k).$$

Total power spectrum reads

$$\Delta_s^2(p) = \Delta_{s(0)}^2(p_*)\left(\frac{p}{p_*}\right)^{n_s-1} \begin{Bmatrix} 1 + \frac{1}{4}(\Delta\eta)^2 \Delta_{s(\mathrm{PBH})}^2 \left(1.1 + \log\frac{k_e}{k_s} + \log\tilde{\Lambda} + \frac{\tilde{\Lambda}^2 - 1}{2}\right) \\ +\mathcal{O}\left[\left((\Delta\eta)^2 \Delta_{s(\mathrm{PBH})}^2\right)^2\right] \end{Bmatrix}$$

where $\tilde{\Lambda} = \Lambda/H$.

Defining

$$\Delta^2_{s(0)}(p_*) \equiv \Delta^2_{s(0)}(p_*, \tilde{\mu}) \left\{ 1 + \frac{1}{4}(\Delta\eta)^2 \Delta^2_{s(0)}(p_*, \tilde{\mu}) \left(\frac{k_e}{k_s}\right)^6 \left(-1.1 - \log\frac{k_e}{k_s} + \log\frac{\tilde{\mu}}{\tilde{\Lambda}} + \frac{\tilde{\mu}^2 - \tilde{\Lambda}^2}{2} \right) + \mathcal{O}\left[\left((\Delta\eta)^2 \Delta^2_{s(\mathrm{PBH})} \right)^2 \right] \right\}$$

with $\tilde{\mu} = \frac{\mu}{H}$, the renormalized power spectrum reads

$$\Delta^2_s(p) = \Delta^2_{s(0)}(p_*, \tilde{\mu}) \left\{ 1 + \frac{1}{4}(\Delta\eta)^2 \Delta^2_{s(0)}(p_*, \tilde{\mu}) \left(\frac{k_e}{k_s}\right)^6 \left(\log\tilde{\mu} + \frac{\tilde{\mu}^2 - 1}{2} \right) + \mathcal{O}\left[\left((\Delta\eta)^2 \Delta^2_{s(\mathrm{PBH})} \right)^2 \right] \right\}$$

At renormalization scale $\mu = H$:

$$\Delta^2_s(p) = \Delta^2_{s(0)}(p_*, \mu = H) \left(\frac{p}{p_*}\right)^{n_s - 1} \left\{ 1 + \mathcal{O}\left[\left((\Delta\eta)^2 \Delta^2_{s(\mathrm{PBH})} \right)^2 \right] \right\}$$

Requirement to renormalize loop correction order by order:

$(\Delta\eta)^2 \Delta^2_{s(\mathrm{PBH})} \ll 1$ $\Longrightarrow$ The same conclusion

Criticism II

2303.00599

A different method, so-called source method, using Maldacena's consistency relation would yield somewhat smaller one-loop correction than what we find using the honest-to-god in-in perturbation theory.

3rd order action for the geometrical variable ζ

$$S^{(3)}[\zeta] = S_{\mathrm{bulk}}[\zeta] + S_{\mathrm{B}}[\zeta] + M_{\mathrm{pl}}^2 \int \mathrm{d}t\, \mathrm{d}^3x\; 2f(\zeta)\left(\frac{\delta L}{\delta\zeta}\right)_1$$

$$g_{ij} = a^2(t)e^{2\zeta}\delta_{ij}$$

$$S_{\mathrm{bulk}}[\zeta] = M_{\mathrm{pl}}^2 \int \mathrm{d}t\, \mathrm{d}^3x\; a^3 \left[\epsilon^2\dot{\zeta}^2\zeta + \frac{1}{a^2}\epsilon^2(\partial_i\zeta)^2\zeta - 2\epsilon\dot{\zeta}\partial_i\zeta\partial_i\chi - \frac{1}{2}\epsilon^3\dot{\zeta}^2\zeta + \frac{1}{2}\epsilon\zeta(\partial_i\partial_j\chi)^2 + \underline{\frac{1}{2}\epsilon\dot{\eta}\dot{\zeta}\zeta^2}\right]$$

Boundary terms

$$S_{\mathrm{B}}[\zeta] = M_{\mathrm{pl}}^2 \int \mathrm{d}t\, \mathrm{d}^3x\, \frac{\mathrm{d}}{\mathrm{d}t}\Big[-9a^3H\zeta^3 + \frac{a}{H}\zeta(\partial_i\zeta)^2 - \frac{1}{4aH^3}(\partial_i\zeta)^2\partial^2\zeta - \frac{a\epsilon}{H}\zeta(\partial_i\zeta)^2$$
$$+\frac{a}{2H^2}\zeta(\partial_i\partial_j\zeta\partial_i\partial_j\chi - \partial^2\zeta\partial^2\chi) - \frac{a^3}{2H^2}\zeta(\partial_i\partial_j\chi\partial_i\partial_j\chi - \partial^2\chi\partial^2\chi) - \frac{\epsilon a^3}{H}\zeta\dot{\zeta}^2 - \frac{\eta a^3}{2}\zeta^2\partial^2\chi\Big]$$

$$\chi = \epsilon\partial^{-2}\dot{\zeta}$$

Last term is proportional to the first-order equation

$$\left(\frac{\delta L}{\delta\zeta}\right)_1 = \frac{\mathrm{d}}{\mathrm{d}t}(\epsilon a^3\dot{\zeta}) - \epsilon a\partial^2\zeta.$$

$$f(\zeta) = \frac{\eta}{4}\zeta^2 + \frac{\dot{\zeta}}{H}\zeta + \frac{1}{4a^2H^2}[-(\partial_i\zeta)^2 + \partial^{-2}\partial_i\partial_j(\partial_i\zeta\partial_j\zeta)] + \frac{1}{2H}[\partial_i\zeta\partial_i\chi - \partial^{-2}\partial_i\partial_j(\partial_i\zeta\partial_j\chi)]$$

Geometrically natural variable ζ is not a proper variable for quantization. It is $\boldsymbol{\zeta}$ defined by $\zeta = \boldsymbol{\zeta} + f(\boldsymbol{\zeta})$ $\boldsymbol{\zeta}=\zeta_n$
with which we find

$$\chi = \epsilon\partial^{-2}\dot{\boldsymbol{\zeta}}$$

$$S^{(2)}[\zeta] = S^{(2)}[\boldsymbol{\zeta}] + \int \mathrm{d}t\ \mathrm{d}^3x \left[(-2)f(\boldsymbol{\zeta})\left(\frac{\delta L}{\delta \boldsymbol{\zeta}}\right)_1 - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a}{2H^2}\boldsymbol{\zeta}(\partial_i\partial_j\boldsymbol{\zeta}\partial_i\partial_j\chi - \partial^2\boldsymbol{\zeta}\partial^2\chi)\right.\right.$$
$$\left.\left. -\frac{a^3}{2H^2}\boldsymbol{\zeta}(\partial_i\partial_j\chi\partial_i\partial_j\chi - \partial^2\chi\partial^2\chi) - \frac{\epsilon a^3}{H}\boldsymbol{\zeta}\dot{\boldsymbol{\zeta}}^2 - \frac{\eta a^3}{2}\boldsymbol{\zeta}^2\partial^2\chi\right)\right]$$

so that $S^{(2)}[\zeta] + S^{(3)}[\zeta] = S^{(2)}[\boldsymbol{\zeta}] + S_{\mathrm{bulk}}[\boldsymbol{\zeta}]$

no boundary terms!

It satisfies $\boldsymbol{\zeta} = -\dfrac{H}{\dot{\phi}}\delta\phi$.

Maldacena (2003)
Arroja Koyama (2008)

$$S[\zeta] = S^{(2)}[\boldsymbol{\zeta}] + S_{\mathrm{bulk}}[\boldsymbol{\zeta}] = M_{\mathrm{pl}}^2 \int \mathrm{d}\tau\ \mathrm{d}^3x\ a^2\epsilon \left[(\boldsymbol{\zeta}')^2 - (\partial_i\boldsymbol{\zeta})^2 + \frac{1}{2}\eta'\boldsymbol{\zeta}'\boldsymbol{\zeta}^2\right]$$

$$\boldsymbol{\zeta}''_{\mathbf{p}} + \frac{(a^2\epsilon)'}{a^2\epsilon}\boldsymbol{\zeta}'_{\mathbf{p}} + \frac{(a^2\epsilon\eta')'}{4a^2\epsilon}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\boldsymbol{\zeta}_{\mathbf{k}}\boldsymbol{\zeta}_{\mathbf{p-k}} = 0$$

source term

$$\zeta''_{\mathbf{p}} + \frac{(a^2\epsilon)'}{a^2\epsilon}\zeta'_{\mathbf{p}} + \frac{(a^2\epsilon\eta')'}{4a^2\epsilon}\int \frac{d^3k}{(2\pi)^3}\zeta_{\mathbf{k}}\zeta_{\mathbf{p}-\mathbf{k}} = 0 \qquad \zeta = \zeta^f + \zeta^s$$

Homogeneous solution without source term $\zeta_p^f(\tau) = \mathcal{A}_p + \mathcal{B}_p \int^{\tau} \frac{d\tau_1}{a^2(\tau_1)\epsilon(\tau_1)}$

Inhomogeneous solution

$$\zeta^s_{\mathbf{p}}(\tau) = -\frac{1}{4}\int_{-\infty}^{\tau}\frac{d\tau_1}{a^2(\tau_1)\epsilon(\tau_1)}\int_{-\infty}^{\tau_1} d\tau_2 \left[a^2(\tau_2)\epsilon(\tau_2)\eta'(\tau_2)\right]'\int\frac{d^3k}{(2\pi)^3}\zeta_{\mathbf{k}}(\tau_2)\zeta_{\mathbf{p}-\mathbf{k}}(\tau_2)$$

Tree $\langle\langle \zeta_{\mathbf{p}}(\tau_0)\zeta_{-\mathbf{p}}(\tau_0)\rangle\rangle_{(0)} \equiv \langle\langle \zeta^f_{\mathbf{p}}(\tau_0)\zeta^f_{-\mathbf{p}}(\tau_0)\rangle\rangle = |\zeta_p(\tau_0)|^2.$

small

One loop $\langle\langle \zeta_{\mathbf{p}}(\tau_0)\zeta_{-\mathbf{p}}(\tau_0)\rangle\rangle_{(1)} \equiv 2\langle\langle \zeta^s_{\mathbf{p}}(\tau_0)\zeta^f_{-\mathbf{p}}(\tau_0)\rangle\rangle + \langle\langle \zeta^s_{\mathbf{p}}(\tau_0)\zeta^s_{-\mathbf{p}}(\tau_0)\rangle\rangle$

$$\langle\langle \zeta^s_{\mathbf{p}}(\tau_0)\zeta^f_{-\mathbf{p}}(\tau_0)\rangle\rangle = \frac{1}{4}\Delta\eta\int\frac{d^3k}{(2\pi)^3}\left[-\underline{\langle\langle \zeta_{\mathbf{k}}(\tau_e)\zeta_{-\mathbf{k}}(\tau_e)\zeta_{\mathbf{p}}(\tau_e)\rangle\rangle} + \frac{2}{3k_e}\langle\langle \zeta'_{\mathbf{k}}(\tau_e)\zeta_{-\mathbf{k}}(\tau_e)\zeta_{\mathbf{p}}(\tau_e)\rangle\rangle\right]$$

Using Maldacena's relation here would yield an incorrect answer as it is a relation for the geometrical variable ζ.

$$\lim_{k_1\to 0}\langle\langle \zeta_{\mathbf{k}_1}(\tau)\zeta_{\mathbf{k}_2}(\tau)\zeta_{-\mathbf{k}_2}(\tau)\rangle\rangle = -(n_s(k_2,\tau)-1)\langle\langle \zeta_{\mathbf{k}_2}(\tau)\zeta_{-\mathbf{k}_2}(\tau)\rangle\rangle\langle\langle \zeta_{\mathbf{k}_1}(\tau)\zeta_{-\mathbf{k}_1}(\tau)\rangle\rangle$$
$$= -(n_s(k_2,\tau)-1)|\zeta_{k_2}(\tau)|^2|\zeta_{k_1}(\tau)|^2, \quad n_s(k,\tau)-1 = \frac{d\log\Delta_s^2(k,\tau)}{d\log k}$$

Proper replacement of $\boldsymbol{\zeta}$ by ζ yields the same answer to ours.

Criticism III 2308.04732 etc

Interplay between the bulk terms and surface terms of the third-order action cancels the one-loop correction,

$$S^{(3)} \supset \int \mathrm{d}\tau\, \mathrm{d}^3\boldsymbol{x} \left[\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{a^2\epsilon}{2}\eta\zeta^2\zeta' + \frac{a\epsilon}{H}\zeta\zeta'^2 \right) \right]$$

so that our argument does not apply.

It would be very nice if this claim could be extended to all order to show entire absence of one-loop corrections. That would be a discovery of new symmetry and much more interesting than what we are claiming! (Note that the Einstein Hilbert action also consists of bulk terms and surface terms.)

Unfortunately, life is not that easy, as calculations using ζ (rather than **ζ**) are not closed at the cubic order; they induce unacceptably large quartic corrections, indicating the necessity to include quartic order action which will cancel them. In the end, complicated calculations using ζ are equivalent with those using **ζ** ?

Loophole?

Smooth transition?

[Comparing sharp and smooth transitions of the second slow-roll parameter in single-field inflation]

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